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Migdal–Kadanoff renormalization group with long-range disorder

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Abstract. Phase transitions in a two-dimensional q-state Pott's model with long-range disorder in which the correlation decays as a power law $\sim r^{-a}$ for separation r are studied analytically within a Migdal–Kadanoff renormalization group. Correlation terms are introduced into the parameter space of the renormalization group and the recursion relations are derived. We expand q near q_0, where the specific-heat exponent of the pure system α_p vanishes, and a near $d = 2$. For small α_p and $\delta = 2 - a \ll 1$, we find three fixed points: 'pure', 'short-range disorder' and 'long-range disorder'. In our calculation the disorder is relevant if $a\nu - 2 < 0$ for $a < d$, where the ν are the correlation-length exponents of the fixed points. The correlation-length exponent for the 'long-range disorder' fixed point is $\nu_{long} = 2/a$. These are consistent with field-theoretic renormalization group results.

1. Introduction

Phase transitions in systems with long-range disorder have been a subject of experimental and theoretical interest over the past decade [1–3]. A simple but useful method for studying the critical properties of disordered systems is the Migdal–Kadanoff renormalization-group approach [4–6]. A lot of work has been undertaken to study disordered systems within the Migdal–Kadanoff approach [3, 7, 8]. The simplest models of disordered systems are defined by the probability distribution of nearest-neighbour interactions. Using the Migdal–Kadanoff method one can write the equation for renormalizing this probability distribution. Though it is an inherently uncontrolled approximation, it often yields qualitative insights and can be directly applied to the physical dimensions. Derrida and Gardner [8] have analytically studied the Potts model on a diamond hierachical lattice with random interactions by weak-disorder expansion. In fact, in their consideration the disorder is short-ranged. Extending their approach to the long-range disorder case we studied the critical phenomena of the random-Potts model on a two-dimensional lattice.

We are concerned with Pott's model on a two-dimensional lattice where the random nearest-neighbour interactions are distributed according to a given probability distribution and the correlation of the interactions falls off with distance as a power law $\sim r^{-a}$. Correlation terms are introduced as the probability distribution is renormalized. The recursion relations of the correlation terms and the moments of the probability distribution are derived. For a small specific-heat exponent of the pure system α_p and $\delta = 2 - a$, we calculate analytically the fixed points and their exponents in powers of α_p and δ. The relevance of the disorder is consistent with the extended Harris criterion [9].

In section 1, we introduce the model. We derive the recursion relations of the moments and the correlation terms of the probability distribution in section 2. In section 3, we calculate the fixed points for small α and δ.

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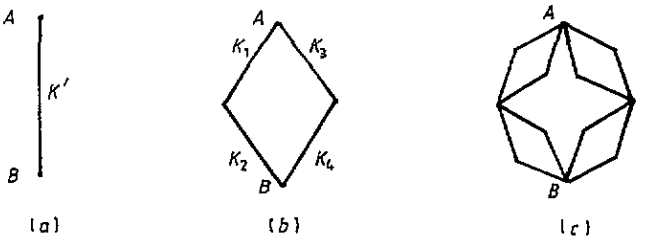

Figure 1. The diamond lattice is constructed recursively. One starts with one bond (*a*). To go from (*a*) to (*b*) one must replace this bond by a set of four bonds. To go from (*b*) to (*c*) one must replace each bond in (*b*) by a set of four bonds, and so on. The order in renormalization transformation is inverted: (*a*) next generation; (*b*) present generation; (*c*) last generation.

2. Definition of the model

The Hamiltonian of the q-state Potts model on a two-dimensional square lattice with long-range-correlated nearest-neighbour interactions is

$$H = -\sum_{\langle i,j \rangle} J_{i,j}\delta_{\sigma_i,\sigma_j} \tag{1}$$

where the spins σ_i can take q different values and the sum runs over all pairs of nearest neighbours. The interactions $J_{i,j}$ are distributed according to a given probability distribution and are long-range correlated in space. It is convenient to work with a variable K on each bond defined as

$$K = e^{\beta J_{ij}}. \tag{2}$$

Accordingly, the variable K is distributed randomly and correlated in space. In our consideration, the correlation of the variable K decays as a power law with the distance

$$\overline{\delta K(r_1)\delta K(r_2)} \sim |r_1 - r_2|^{-a} \tag{3}$$

where $a > 0$ and $\delta K(r_1) = K(r_1) - \overline{K(r_1)}$ and $\overline{\ldots}$ represents the average of '...' over the space.

Migdal's recurrence relation on a d-dimensional lattice may be derived by dividing the system into hypercubes of b^d spins, moving all the bonds onto the edge of the hypercubes and then decimating all except the corner spins [6]. A hypercube may be called a cell. After renormalization the lattice retains the geometrical symmetry and the cell's size is extended b times. By rescaling, the cell's size can be taken to be of unit length.

The Migdal–Kadanoff approach is equivalent to an exact decimation on a hierachical lattice [10]. The decimation on a diamond hierachical lattice, which is often used to study disordered systems, is shown in figure 1. From the present generation to the next, each set of four bonds K_1, K_2, K_3, K_4 can be replaced by one single bond K' and the renormalization transformation is [8]

$$K' = (\{K_i\}) = \left(\frac{K_1K_2 + q + 1}{K_1 + K_2 + q - 2}\right)\left(\frac{K_3K_4 + q - 1}{K_3 + K_4 + q - 2}\right). \tag{4}$$

After a decimation, the probability distribution $P(K')$ for the next generation can be deduced from $P(K)$ at the present generation

$$P(K') = \int \prod_{i=1}^{4} \mathrm{d}K_i \, P(\{K_i\})\delta(K' - R(\{K_i\})) \tag{5}$$

where $P(\{K_i\})$ is the united distribution since the bonds $\{K_i\}$ are correlated. The united distribution of any two bonds K' at the next generation is

$$P(K'_1, K'_2) = \int P(\{K_i^{(1)}\}, \{K_i^{(2)}\})\delta(K'_1 - R(\{K_i^{(1)}\}))\delta(K'_2 - R(\{K_i^{(2)}\})) \prod_{j=1}^{2}\prod_{i=1}^{4} \mathrm{d}K_i^{(j)} \tag{6}$$

where the superscript 'j' labels the jth cell at the present generation. One can study the critical behaviour of disordered systems according to transformations (4)–(6).

Here we give some results for a pure system [8] which will be used in the following. For a given value of q, the critical point is

$$K_c = R(K_c) = \left(\frac{K_c^2 + q - 1}{2K_c + q - 2}\right)^2 \tag{7}$$

which turns out to be $q = (\sqrt{K_c} - 1)(K_c - 1)$ and the specific-heat exponent is given by

$$\alpha_p = 2 - \frac{\log 4}{\log R'(K_c)} \tag{8}$$

The value of q when α_p vanishes is $q_0 = 4 + 2\sqrt{2}$.

3. Recursion relations

We studied the renormalization transformations (5) and (6), by considering a narrow distribution concentrated around the fixed point K_c. By writing

$$K_i = K_c + \varepsilon_i \qquad K' = K_c + \varepsilon' \tag{9}$$

one can expand ε' in powers of ε_i using formula (4). We have expanded ε' up to the fourth power and the result is

$$\begin{aligned}\varepsilon' = {} & 4A\sqrt{K_c}\varepsilon_i - 4AB\sqrt{K_c}\varepsilon_i^2 + (2B\sqrt{K_c} - 4AB\sqrt{K_c} + 4A^2)\varepsilon_i\varepsilon_j \\ & + (12AB^2\sqrt{K_c} - 4B^2 - 8A^2B)\varepsilon_i\varepsilon_j^2 + 4AB^2\sqrt{K_c}\varepsilon_i^3 + (4AB - 8A^2B)\varepsilon_i\varepsilon_j\varepsilon_k \\ & + (4B^3\sqrt{K_c} - 12AB^3\sqrt{K_c} + 4A^2B^2)\varepsilon_i^2\varepsilon_j^2 - (12AB^2 - 32A^2B^2)\varepsilon_i\varepsilon_j\varepsilon_k^2 \\ & + (4B^2\sqrt{K_c} - 16AB^3\sqrt{K_c} + 8A^2B^2)\varepsilon_i\varepsilon_j^3 + (B^2 - 4AB^2 + 4A^2B^2)\varepsilon_i\varepsilon_j\varepsilon_k\varepsilon_l \\ & - 4AB^3\sqrt{K_c}\varepsilon_j^4\end{aligned} \tag{10}$$

where A, B are given by

$$A = \sqrt{K_c}/(\sqrt{K_c}+1)^2 \qquad B = 1/[(K_c-1)(\sqrt{K_c}+1)]$$

and $i, j, k, l = 1, 2, 3, 4$ and $i \neq j \neq k \neq l$. These subscripts label the different bonds in a cell rather than summation. The repeat times are taken into account in the coefficients. The powers of ε', such as $\varepsilon'^2, \varepsilon'^3, \ldots$, can also be expanded in powers of ε_i. We do not give these expressions explicitly here because they are rather long and complicated and can be obtained directly.

The moments of the probability $P(K')$ are obtained by averaging the powers of ε', where one finds multibody terms such as $\overline{\varepsilon_i\varepsilon_j}, \overline{\varepsilon_i\varepsilon_j\varepsilon_k}$. Writing

$$\varepsilon_i = \overline{\varepsilon} + \delta\varepsilon_i \tag{11}$$

one has

$$\overline{\varepsilon_i\varepsilon_j} = \overline{\varepsilon}^2 - \overline{\delta\varepsilon_i\delta\varepsilon_j}. \tag{12}$$

We need to treat the correlation terms between bonds in a cell such as $\overline{\delta\varepsilon_i\delta\varepsilon_j}$.

One can see that on the original lattice the correlation of ε_i takes the same form as in (3)

$$\overline{\delta\varepsilon(r_1)\delta\varepsilon(r_2)} \sim |r_1 - r_2|^{-a}. \tag{13}$$

Since the separations between bonds in a cell are about a cell's size, we define a variable

$$\Delta_1 = \overline{\delta\varepsilon(r_1)\delta\varepsilon(r_2)} \qquad (r_1 - r_2 = \text{a cell's size}) \tag{14}$$

at each generation. In fact Δ_1 is the coefficient of the right-hand side of equation (13) as the cell's size is scaled to be a unit length. Thus we have $\overline{\delta\varepsilon_i\delta\varepsilon_j} = \Delta_1$ in equation (12). Similarly, we can define other correlation terms with high powers of ε. In our calculation we use two other variables Δ_2 and Δ_3 defined by

$$\Delta_2 = \overline{\delta\varepsilon(r_1)\delta\varepsilon^2(r_2)} \qquad \Delta_3 = \overline{\delta\varepsilon(r_1)\delta\varepsilon^3(r_2)} \qquad (r_1 - r_2 = \text{a cell's size}). \tag{15}$$

The average of the multibond terms, such as $\overline{\varepsilon_i\varepsilon_j\varepsilon_k}$, can be treated according to definition (11) and the approximation that the distances between bonds in a cell are the same and equal to one, that is $\overline{\varepsilon_i\varepsilon_j\varepsilon_k} = \overline{\varepsilon}^3 + 3\overline{\varepsilon}\Delta_1 + \overline{\delta\varepsilon_i\delta\varepsilon_j\delta\varepsilon_k}$. The last term will be discussed in the following.

Now the parameter space is extended in comparison with the short range case [8]. In order to obtain the recursion relations for the correlation terms, we consider the correlation of ε' after a step of renormalization at two different positions r_1 and r_2

$$\overline{\delta\varepsilon'(r_1)\delta\varepsilon'(r_2)} = \overline{\varepsilon'(r_1)\varepsilon'(r_2)} - \overline{\varepsilon'}^2.$$

Substituting equation (4) into this equation yields

$$\begin{aligned}\overline{\delta\varepsilon'(r_1)\delta\varepsilon'(r_2)} &= 16A^2K_c\overline{\delta\varepsilon(r_1)\delta\varepsilon(r_2)} - 32A^2BK_C\overline{\delta\varepsilon(r_1)\delta\varepsilon^2(r_2)} \\ &\quad + 8A\sqrt{K_c}(2B\sqrt{K_c} - 4AB\sqrt{K_c} + 4A^2)\overline{\delta[\varepsilon_i(r_1)\varepsilon_j(r_1)]\delta\varepsilon(r_2)} \\ &\quad + (2B\sqrt{K_c} - 4AB\sqrt{K_c} + 4A^2)^2\overline{\delta[\varepsilon_i(r_1)\varepsilon_j(r_1)]\delta[\varepsilon_k(r_2)\varepsilon_l(r_2)]} + \cdots \end{aligned} \tag{16}$$

where we have included only some typical terms which need to be discussed.

On the original lattice the correlation of ε decays as a power law with the separation. It is expected that the correlation of ε' after renormalization retains the same decaying form. From the definition of Δ_1 and equation (13) one can see that the first term on the right-hand side of equation (16) is

$$\overline{\delta\varepsilon(r_1)\delta\varepsilon(r_2)} = \frac{\Delta_1}{|r_1 - r_2|^{-a}}. \tag{17}$$

The third and fourth terms on the right-hand side of equation (16) are

$$\overline{\delta[\varepsilon_i(r_1)\varepsilon_j(r_1)]\delta\varepsilon(r_2)} = \frac{2\bar{\varepsilon}\Delta_1}{(r_1 - r_2)^a} + \overline{\delta\varepsilon_i(r_1)\delta\varepsilon_j(r_1)\delta\varepsilon(r_2)} \tag{18}$$

$$\begin{aligned}\overline{\delta[\varepsilon_i(r_1)\varepsilon_j(r_1)]\delta[\varepsilon_k(r_2)\varepsilon_l(r_2)]} &= \frac{4\bar{\varepsilon}^2\Delta_1}{(r_1 - r_2)^a} + 4\bar{\varepsilon}\overline{\delta\varepsilon_i(r_1)\delta\varepsilon_j(r_1)\delta\varepsilon_k(r_2)} \\ &\quad - \Delta_1^2 + \overline{\delta\varepsilon_i(r_1)\delta\varepsilon_j(r_1)\delta\varepsilon_k(r_2)\delta\varepsilon_l(r_2)}.\end{aligned} \tag{19}$$

For a general Gaussian distribution, which can describe random variables in many natural systems, the average of multi-variables such as $\overline{\delta\varepsilon_i(r_1)\delta\varepsilon_j(r_1)\delta\varepsilon(r_2)}$ and $\overline{\delta\varepsilon_i(r_1)\delta\varepsilon_j(r_1)\delta\varepsilon_k(r_2)\delta\varepsilon_l(r_2)}$ can be obtained from Wick's theorem [11]. All such terms with an odd number of factors are zero and those with an even number are given by the sum of the products of all possible pairs. In our case

$$\overline{\delta\varepsilon_i(r_1)\delta\varepsilon_j(r_1)\delta\varepsilon(r_2)} = \overline{\delta\varepsilon_i(r_1)\delta\varepsilon_j(r_1)\delta\varepsilon_k(r_1)} = 0 \tag{20}$$

$$\begin{aligned}\overline{\delta\varepsilon_i(r_1)\delta\varepsilon_j(r_1)\delta\varepsilon_k(r_2)\delta\varepsilon_l(r_2)} &= \overline{\delta\varepsilon_i(r_1)\delta\varepsilon_j(r_1)}\ \overline{\delta\varepsilon_k(r_2)\delta\varepsilon_l(r_2)} \\ &\quad + \overline{\delta\varepsilon_i(r_1)\delta\varepsilon_k(r_2)\delta\varepsilon_j(r_1)\delta\varepsilon_l(r_2)} + \overline{\delta\varepsilon_i(r_1)\delta\varepsilon_l(r_2)\delta\varepsilon_j(r_1)\delta\varepsilon_k(r_2)} \\ &= \Delta_1^2 + 2\frac{\Delta_1^2}{(r_1 - r_2)^{2a}}.\end{aligned} \tag{21}$$

As can be seen from the right-hand side of equation (19), some terms retain the decaying form r^{-a} and some do not. Although this is a bit disturbing, it can be overcome. We neglect the terms with higher-decaying exponents such as r^{-2a} in equation (21) because it falls off faster than r^{-a}. The first term in equation (21) is cancelled in equation (19)

According to definition (14), Δ_1' is equal to the correlation of two bonds K' with a separation of a cell's size at the next generation, therefore, we can obtain Δ_1' by letting $|r_1 - r_2| = b = 2$ in equation (16) where we think that the distances between bonds in two neighbouring cells are approximately the same and equal to $b = 2$. Although the approximation is rough, it is plausible since critical properties do not depend on fluctuation over small scales. We expect that it can keep important information about the critical problem. Thus we have

$$\begin{aligned}\Delta' = \frac{1}{2^a}\{&16A^2K_c\Delta_1 - 32A^2BK_c\Delta_2 + 16A\sqrt{K_c}(2B\sqrt{K_c} - 4AB\sqrt{K_c} + 4A^2)\bar{\varepsilon}\Delta_1 \\ &+ 4(2B\sqrt{K_c} - 4AB\sqrt{K_c} + 4A^2)^2\bar{\varepsilon}^2\Delta_1 + \cdots\}.\end{aligned} \tag{22}$$

Now, all the terms in the recursion relations are well defined and can be obtained directly.

In order to get a truncation of the recursion relations, one can multiply each cumulant and correlation term by some power of a small number λ and retain terms of a given order at each step in the calculation. Of course, the truncation scheme should be self-consistent. We refer the reader to [8] for a more detailed discussion of the truncation scheme.

4. Fixed points for small α_p and δ

In [8], the authors consider the cases for small α_p and study the relevance of the disorder. The sign of the specific-heat exponent of a pure system α_p determines the stability of the pure fixed point. Weinrib and Halperin [9] have studied a long-range disordered model using scaling arguments and a field-theoretic renormalization group. They showed that for $a < d$, the long-range disorder will lead to a new requirement for the stability of the pure fixed point. Therefore, we study the relevance of the disorder in our model for small α_p and a near $d = 2$.

For a value of q which is close to $q_0 = 4 + 2\sqrt{2}$, i.e. $q - q_0 \ll 1$, the specific-heat exponent of the pure system α_p and $K_c(q)$ satisfy (see equations (7) and (8))

$$K_c(q) - K_c(q_0) = \tfrac{1}{2}(10 + 7\sqrt{2})\alpha_p \ln 2. \tag{23}$$

For a near 2, one can introduce a small number $\delta = 2 - a$.

We consider the cases $\alpha_p \sim \delta \sim \lambda$, where λ is a small number. For small α_p and δ we find three fixed points for which the cumulants are in powers of α_p and δ. We calculate the fixed points and their exponents to the λth order. A self-consistent truncation of the recursion relations is given by

$$\overline{\varepsilon'} = (2 + \alpha_p \ln 2)\overline{\varepsilon} + \tfrac{1}{2}(4 - 3\sqrt{2})\overline{\varepsilon^2} + (6 + 4\sqrt{2})\overline{\varepsilon}^2 + (6 + 4\sqrt{2})\Delta_1 + \tfrac{1}{4}(113 - 80\sqrt{2})\overline{\varepsilon}\,\overline{\varepsilon^2} + \tfrac{1}{16}(734 - 519\sqrt{2})\overline{\varepsilon^2}^2 + \tfrac{1}{4}(17 - 12\sqrt{2})\overline{\varepsilon^3} + \tfrac{1}{16}(140 - 99\sqrt{2})\overline{\varepsilon^4} \tag{24}$$

$$\overline{\varepsilon'^2} = (1 + \alpha_p \ln 2)\overline{\varepsilon^2} + 3\overline{\varepsilon}^2 + 3\Delta_1 + \tfrac{1}{2}(36 - 25\sqrt{2})\overline{\varepsilon}\,\overline{\varepsilon^2} + \tfrac{1}{8}(379 - 268\sqrt{2})\overline{\varepsilon^2} + \tfrac{1}{2}(4 - 3\sqrt{2})\overline{\varepsilon^3} + \tfrac{1}{8}(51 - 36\sqrt{2})\overline{\varepsilon^4} \tag{25}$$

$$\overline{\varepsilon'^3} = \tfrac{9}{2}\overline{\varepsilon}\,\overline{\varepsilon^2} + \tfrac{1}{8}(108 - 75\sqrt{2})\overline{\varepsilon^2}^2 + \tfrac{1}{2}\overline{\varepsilon^3} + \tfrac{1}{8}(12 - 9\sqrt{2})\overline{\varepsilon^4} \tag{26}$$

$$\overline{\varepsilon'^4} = \tfrac{9}{2}\overline{\varepsilon^2}^2 + \tfrac{1}{4}\overline{\varepsilon^4} \tag{27}$$

$$\Delta_1' = [1 + (\alpha_p + \delta)\ln 2]\Delta_1 + \tfrac{1}{2}(4 - 3\sqrt{2})\Delta_2 + (12 - 8\sqrt{2})\overline{\varepsilon}\Delta_1 + \tfrac{1}{4}(113 - 80\sqrt{2})\overline{\varepsilon^2}\Delta_1 + \tfrac{1}{4}(17 - 12\sqrt{2})\Delta_3 + c_1\Delta_1^2 \tag{28}$$

$$\Delta_2' = \tfrac{1}{2}\Delta_2 + 3\overline{\varepsilon}\Delta_1 + \tfrac{1}{4}(36 - 25\sqrt{2})\overline{\varepsilon^2}\Delta_1 + \tfrac{1}{4}(4 - 3\sqrt{2})\Delta_3 + c_2\Delta_1^2 \tag{29}$$

$$\Delta_3' = \tfrac{1}{4}\Delta_3 + \tfrac{9}{4}\overline{\varepsilon^2}\Delta_1 + c_3\Delta_1^2 \tag{30}$$

where c_1, c_2, c_3 are constants. In order to solve the long-range-disorder fixed point the terms of Δ_1^2 are needed; however, their coefficients are not important in our calculation. The fixed points are obtained by letting $\overline{\varepsilon'^n} = \overline{\varepsilon^n}$ and $\Delta_i' = \Delta_i$. We find three fixed points: 'pure', 'short-range disorder' and 'long-range disorder'. The 'pure' fixed point is

$$\overline{\varepsilon^n} = 0 \qquad \Delta_i = 0. \tag{31}$$

The 'short-range disorder' fixed-point is

$$\overline{\varepsilon} = \tfrac{1}{7}(44 + 31\sqrt{2})\alpha_p \ln 2 \qquad \overline{\varepsilon^2} = \tfrac{1}{7}(362 + 256\sqrt{2})\alpha_p \ln 2$$

$$\overline{\varepsilon^3} \sim \overline{\varepsilon^3} \sim O(\lambda^2) \qquad \Delta_i = 0 \tag{32}$$

and the 'long-range disorder' fixed-point is

$$\overline{\varepsilon} = \tfrac{1}{2}(10+7\sqrt{2})(\delta+\alpha_p)\ln 2 \qquad \overline{\varepsilon^2} = (41+29\sqrt{2})(\delta+\alpha_p)\ln 2$$

$$\overline{\varepsilon^3} \sim \overline{\varepsilon^4} \sim \Delta_1 \sim O(\lambda^2). \tag{33}$$

Taking $\overline{\varepsilon^n}$ and Δ_i as the components of a vector $\{v_i\}$, the exponents of the fixed points can be obtained from the eigenvalues of the matrix whose elements are

$$M_{kl} = \left.\frac{\partial v_k'}{\partial v_l}\right|_{v=v^*} \tag{34}$$

where v^* is the position of the fixed point. The three maximum eigenvalues of each fixed point are given in table 1.

Table 1. The three maximum eigenvalues of the fixed points.

Fixed points	λ_1	λ_2	λ_3
Pure	$2+\alpha_p \ln 2$	$1+\alpha_p \ln 2$	$1+(\alpha_p+\delta)\ln 2$
Short	$2+(\ln 2/7)(1-2\sqrt{2})\alpha_p$	$1-\alpha_p \ln 2$	$1+[\delta+\frac{1}{7}(1-2\sqrt{2})\alpha_p]\ln 2$
Long	$2-\delta \ln 2$	$1-[(2-\sqrt{2})\alpha_p+(3-\sqrt{2})\delta]\ln 2$	1

The exponents of a fixed point are given by

$$\nu_i = \frac{\ln 2}{\ln \lambda_i}. \tag{35}$$

The correlation-length exponent for a fixed point is the maximal exponent. If any exponents of a fixed point, except the thermal one, are negative they will control the critical behaviour. We call it a stable fixed point. A summary of the regions where the various types of critical behaviour occur, for small α_p and δ, is given in figure 2.

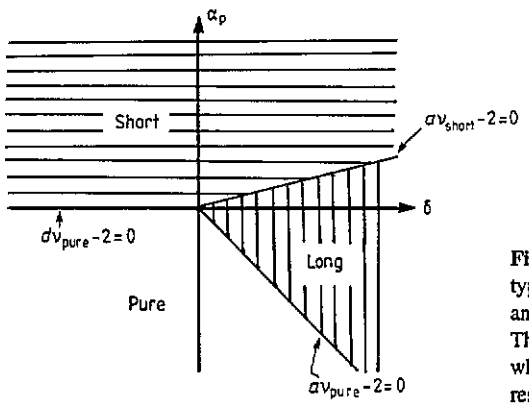

Figure 2. Regions in the δ–α_p plane where various types of critical behaviour occurs. Here $\delta = 2-a$ and α_p is the specific heat exponent of the pure system. The crossover occurs when $a\nu - 2$ becomes negative, where ν is the correlation-length exponent, ν_{pure}, ν_{short} respectively.

From table 1, one can see that there exists a crossover from the 'pure' fixed point to the 'short-range disorder' fixed point when $\alpha_p > 0$ and the crossover exponent is α_p. We recover Derrida's results for the short-range disorder case [8]. For $\delta > 0$, the 'pure' fixed point is unstable when

$$\alpha_p + \delta = (2/\nu_{pure}) - a > 0 \tag{36}$$

and the 'long-range disorder' fixed point is unstable when

$$\delta + \tfrac{1}{7}(1 - 2\sqrt{2})\alpha_p = (2/\nu_{short}) - a > 0 \tag{37}$$

to the order λ. That is, for $a < d$, the long-range disorder is irrelevant if

$$a\nu - 2 > 0 \tag{38}$$

where ν is the correlation-length exponent of the fixed points 'pure' and 'short-range disorder'. For $\alpha_p + \delta > 0$, the 'long-range disorder' fixed point is physically and marginally stable (note $\lambda_3 = 1$ for the long-range-disorder fixed point in our calculation). Its correlation-length exponent is

$$\nu_{long} = \frac{\ln 2}{\ln(2 - \delta \ln 2)} = \frac{2}{a} + O[\lambda^2]. \tag{39}$$

This is consistent with scaling arguments and field-theoretic results [9].

Since the Migdal–Kadanoff approach can be applied to the physical dimension, it would be interesting to use the model described in this paper to study the three-dimensional case.

Acknowledgments

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